

V. STOCHASTIC PROCESS

MARKOV CHAINS

* Stochastic is a Greek word which means that "random" (or) "chance".

Stochastic Process :- It is defined as a collection of random variables.

$$[x(t_n) : n=1, 2, 3, \dots]$$

The random variable $x(t)$ stands for the observation at time 't'.

The no. of states 'n' may be finite (or) infinite depending upon the - time range.

* Classification of Stochastic process:-

1. If both x & t are continuous the stochastic process is called as "continuous stochastic process".

2. If x is continuous & t is discrete, the stochastic process is called as a "Discrete stochastic process".

3. If x is discrete & t is continuous, the stochastic process is called as "discrete stochastic process".

4. If both x & t are discrete, then the stochastic process is called a "Discrete stochastic process".

an illustration for the state and transition probability matrix for instant $t = 0$ onwards unit-5 pg-1/19

→ We can classify stochastic process in another way also :~

* A random process is called a "deterministic stochastic process". if all the future values can be predicted from past observations.

* A stochastic process is called a "non-deterministic stochastic process". if future values of any sample function be predicted from past observation.

Examples :~

1. A queuing system : In this system, the no. of persons joining the queue, the no. of persons being serviced in a time interval & the no. of persons in the queue at time t are all random variables depending on the parameters of the system.

2. Turbulent fluid flow : In this, the velocity components u, v, w of the fluid are random variables depending on the space co-ordinates (x, y, z) & the time t .

3. Movement of molecules of a gas or liquid : In this, at random instants the molecule collides with other molecules. Thus its velocity & position are altered. Thus the state of the molecule is subjected to random change at every instant of time.

(2)

4. A random walk model :- A particle move in a line straight line in steps of unit length. At each stage it can move one step to the right with the probability 'p' (or) One step to the left with the probability 'q' where $p+q=1$.

If the particle from origin its position after n moments is a random variable, which depends on discrete parameter n .

5. Communication Process :- The amplitude of the signals to the transmitted amplitude of the noise produced in the channel depending on time are both random variables.

* Explain about stationary Random Process :-

A random process whose statistical properties do not change with time 't'. Then the random process is called as "stationary random process".

- A random process whose statistical properties change with time 't'. Then the random process is called as "non-stationary random process".
- stationary process is used in "Time series analysis".

Let $x(t)$ be a random variable $x(t_0)$ is considered and its mean is $E[x(t_0)]$.

$$\text{If } E[x(t_1)] = E[x(t_2)],$$

i.e. if the start value of random process $x(t)$

is same at $t=t_1$ & $t=t_2$, its mean is not

affected by a time shift (t sec).

* In an the statistical properties of $x(t)$ are not affected by time shift is called "strong sense" or strict sense stationary random process".

* A random process is said to be "weak series or wide sense stationary random process".

If the mean of the process $x(t)$ is

independent of time i.e. $E[x(t)] = \text{Constant}$.

* Explain classification of states:-

There are 3 types of states:-

1. Transient state :- A state is said to be

Transient state. If it is possible to leave the state and never return.

2. Periodic state :- A state is said to be periodic

state. If the state is return to any one

multiplies of some 've integer other than

'1'. This integer is called "period" of a state.

3. Ergodic state :- A state is said to be ergodic

state if it is neither transferred nor

Periodic.

$$[(at)x]_0 = [(ax)^t]_0 \quad \text{if units, pg - 4/9}$$

* Explain about Ergodic Process?

A random process $X(t)$ is said to be mean ergodic if it's most probable state is equal to its time avg.

i.e :- $E[X(t)] = \bar{x}(t)$ with probability of 1.

⇒ Markov Process in stochastic (random) system is called

a markov process if the occurrence of a future state depends on the immediately preceding state & only on it.

Thus if $x(t_0), x(t_1), \dots, x(t_n)$ represents the points in time scale, then the family of random variables $\{x(t_n)\}$ is said to be a markovian property.

$$P[x(t_n) = x_n | x(t_{n-1}) = x_{n-1}, \dots, x(t_0) = x_0]$$

$$P[x(t_n) = x_n | x(t_{n-1}) = x_{n-1}]$$

Markov process is a sequence of 'n' experiments in which each experiment has 'n' possible outcomes of the preceding experiment.

characteristics of markov process ~

If it is based on the following characteristics

- (i) The states are both collectively exhaustive & mutually exclusive.
- (ii). The problem must have a finite no. of states.

(iii). The transition probability are stationary.

(iv) The probability of moving from one state to another state depends only on the immediately proceeding state.

(v). The transition probability of moving two alternative states in the next time period given a state in the current time period must sum up to unity (1).

⇒ Transition Probability is the probability of initial moving from one state to another (or) remaining in the same state during a single time period is called "transition probability".

Mathematically, the probability:

$$P_{x_{n+1}=x_n} = P[x(t_n)=x_n | x(t_{n-1})=x_{n-1}]$$

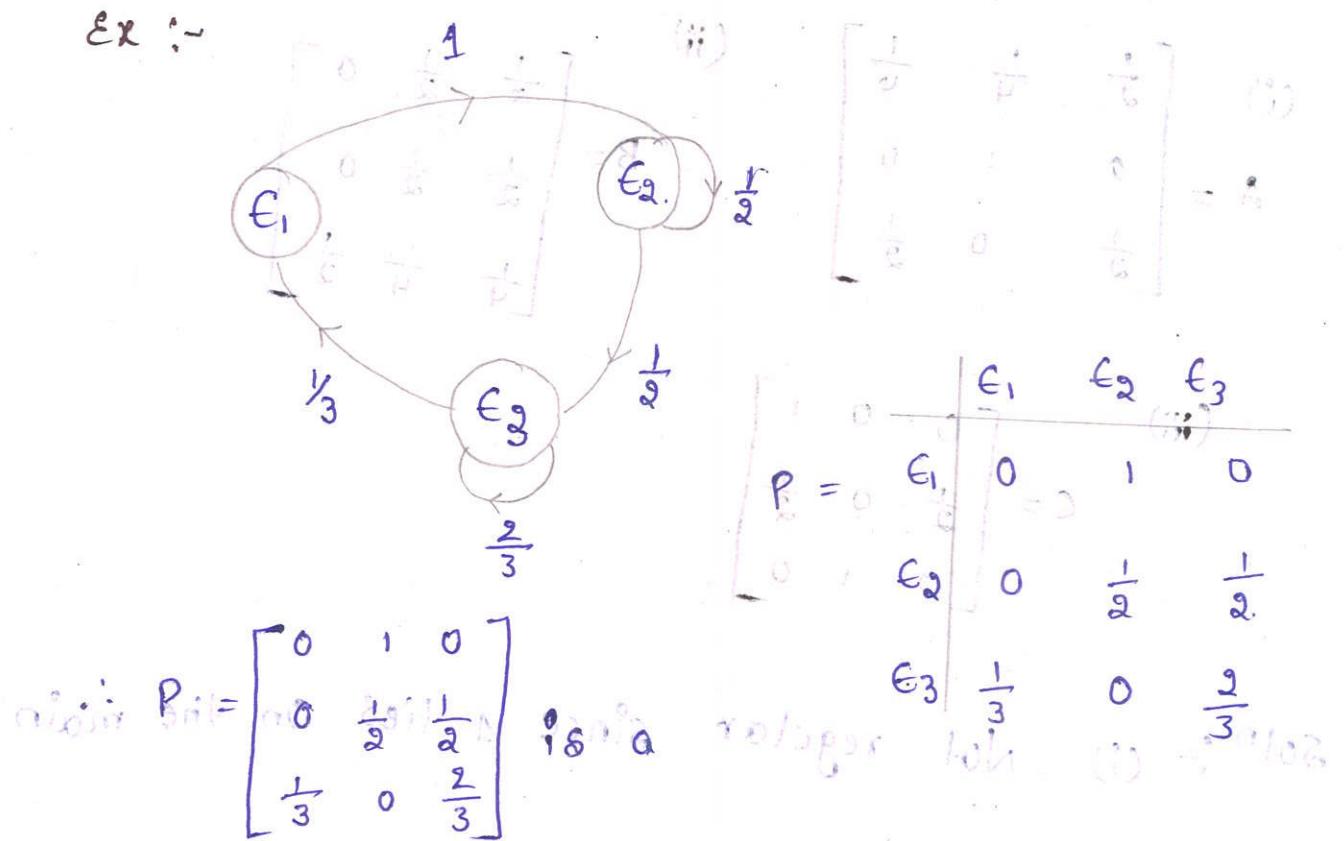
is called the "transition Probability".

⇒ Transition Probability matrix is The transition Probability (P_{ij}) can be arranged in a matrix form. Such matrix is called as a "transition Probability matrix".

$$P = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1m} \\ P_{21} & P_{22} & \dots & P_{2m} \\ \vdots & & & \\ P_{m1} & P_{m2} & \dots & P_{mm} \end{bmatrix}$$

The matrix P is a square matrix whose each element is non-negative (positive) & sum of elements of each row is unity (1). Transition matrix which gives the complete description of the markov process.

Ex :-



Given $P_{ij} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix}$ is a stochastic matrix.

① Test the following matrices are stochastic or not.

$$\text{(a)} \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{1}{2} & 1 & \frac{1}{2} \end{bmatrix} \quad \text{(b)} \begin{bmatrix} \frac{15}{16} & \frac{1}{16} \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

SOLN :- (a) Given matrix is not a square matrix
 \therefore It is a stochastic matrix.

(b) The matrix is a square matrix with non-negative entries. But sum of elements in each row is not equal to 1.

\therefore The matrix is not stochastic. unit-5_pg-7/19

(C). The given matrix is square matrix with non-negative entries & sum of the elements in each row is equal to 1. So it is a stochastic matrix.

Q2 Which of the stochastic matrices are regular.

$$(i) A = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$(ii) B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$$(iii) C = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$$

Soln :- (i) Not regular since 1 lies on the main diagonal.

$$B^2 = B \cdot B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{4} \end{bmatrix}$$

$$B^3 = B^2 \cdot B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{7}{16} & \frac{7}{16} & \frac{1}{8} \end{bmatrix}$$

Since entries b_{13}, b_{23} are zero, B is not regular

$$(iii) C^2 = C \cdot C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$C^3 = C^2 \cdot C = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{7}{16} & \frac{7}{16} & \frac{1}{8} \end{bmatrix}, \quad C^4 = C^3 \cdot C = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$C^5 = C^4 \cdot C = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{3}{8} & \frac{1}{2} & \frac{3}{8} \\ \frac{1}{8} & \frac{1}{2} & \frac{3}{8} \end{bmatrix}$

Since all the entries of C^5 are positive, C is a regular stochastic matrix.

Q3 Which of the following matrices are regular?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 1 \\ 0 & \frac{3}{4} & 1 \\ 0 & 0 & 1 \end{bmatrix}$

SOLN :- In all the matrices, the principal diagonal contains 1. Thus they are not regular.

Q4 A training process is considered as a 2-state Markov Chain. If it rains, it is considered to be in state 0, & if it does not rain, the chain is in the state of 1. The transition probability of the markov chain is defined by $P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$.

Find the probability that it will rain for 3 days from today assuming that it is raining today. Assume that mutual probabilities of state 0 (or) state 1 is 0.4 & 0.6 respectively.

SOLN :- The 1st step transition probability of the matrix is given by $P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$

$$P(2) = P^2 = \begin{bmatrix} 0.44 & 0.56 \\ 0.28 & 0.72 \end{bmatrix}$$

$$P(3) = P^3 = \begin{bmatrix} 0.376 & 0.624 \\ 0.312 & 0.688 \end{bmatrix}$$

The probability that it will rain on third day given that it will rain today is 0.376.

\Rightarrow classification of states & chains:

The state of a markov chain $\{X_n\}_{n \geq 0}$:

Can be classified in a distinctive manner. According to some fundamental properties of the system.

* If every state can be reached from any state

then the chain is said to be irreducible. Then

the transition matrix is irreducible. Or non-

reducible. Otherwise the chain is said to be

reducible (or) non-reducible.

* A state i is said to be an absorbing state

if and only if $P_{ii} = 1$. A Markov chain is said to

absorbing state if it has at least one absorbing

state & it is possible to go from every other

non-absorbing state to atleast one absorbing

state in one or more steps. i.e. in n steps

* A state i of a Markov chain is called a

"return state" if $P_{ij}^{(n)} \geq 1$ for some $n \geq 1$.

* A state i is said to be periodic with period $t(≥ 1)$ if the return to that state is possible only after t steps, where t is the greatest integer with this property.

In this case $P_{ij}^{(n)} = 0$, unless n is an integral multiple of t .
 \rightarrow The state i is said to be a periodic (or non-periodic) if no such $t(≥ 1)$ exists.

* A positive recurrent (or positive persistent) & a periodic state is called ergodic. A Markov chain all of whose states are ergodic is said to be a ergodic chain.

* A tpm, P is said to be a regular matrix if all entries of P^m , ($m=2, 3, \dots$) are non-zero positive values. A homogeneous markov chain is said to be a regular chain if its tpm is a regular matrix.

\rightarrow A tpm, P is said to be a stochastic matrix if the elements of each of the rows are non-negative & the sum of elements in each row is equal to 1.

Unit 5 and out, syllabus of marks will be

Cell of A	Cell of B
Cell of C	Cell of D
Cell of E	Cell of F

Cell of G	Cell of H
Cell of I	Cell of J
Cell of K	Cell of L

① Three boys A, B, C are throwing a ball to each other. A always throws the ball to B, and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix

& classify the states: Do all the states are ergodic?

Soln: The transition probability matrix of the states (left) drawn to (right)

Process $\{x_n\}$ is given as

\rightarrow

Need to show if states of x_n are disjoint
 \downarrow states of x_{n-1}

$$P = \begin{bmatrix} A & B & C \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

states of x_n depends only on states of

x_{n-1} but not on states of x_{n-2}, x_{n-3}, \dots for earlier states it is only relevant to A, B or

hence $\{x_n\}$ is a Markov chain.

$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, P^3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$$P_{12}^{(3)} > 0, P_{13}^{(2)} > 0, P_{21}^{(2)} > 0, P_{23}^{(2)} > 0, P_{33}^{(2)} > 0 \text{ and}$$

$$\text{all other } P_{ij}^{(1)} > 0$$

\therefore The chain is irreducible, we can see that

$$P^4 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$$P^5 = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{2} \end{bmatrix}$$

$$P^6 = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{2} \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} \end{bmatrix}$$

(7)

reduces to next state
and so on.

We note that $P_{ii}^{(2)}, P_{ii}^{(3)}, P_{ii}^{(5)}, P_{ii}^{(6)}$ etc. are > 0
for $i=2,3$.

$$\text{G.C.D of } 3, 5, 6, \dots = 1.$$

∴ The state A (i.e state A) is Periodic with Period 1
i.e. a periodic.

Since the chain is finite & irreducible (all its states
are non-null to persistent) & all the states are

(2) A gambler has Rs. 2. He bets Rs. 1 at a time
and wins Rs. 1 with probability $\frac{1}{2}$. He stops
playing if he loses Rs. 2 or wins Rs. 4.

(a) What is the related markov chain?

(b) What is the probability that he has lost his
money at the end of 5 plays?

(c) What is the probability that the game lasts
more than 7 plays?

Sol: Let the (x_n) represent the amount with
the player at the end of the n th round
of the play.

The state space of $x_n = \{0, 1, 2, 3, 4, 5, 6\}$

When the game is stopped if the player loses Rs. 2.

$x_n = 0$ (or) if he wins Rs. 4, $x_n = 6$.

The tpm of the markov chain is written as

	0	1	2	3	4	5	6	7	8	9
0	1	0	0	0	0	0	0	0	0	0
1	0	1/3	0	1/3	0	1/3	0	0	0	0
2	0	0	1/3	0	1/2	0	0	0	0	0
3	0	0	0	1/2	0	1/2	0	0	0	0
4	0	0	0	0	1/2	0	1/2	0	0	0
5	0	0	0	0	0	1/2	0	1/2	0	0
6	0	0	0	0	0	0	1/2	0	1/2	0
7	0	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	1

Note: We can notice that this is a random walk with absorbing barriers at 0 & 86, since the chain cannot come out of the states 0 and 86, once it has entered.

(b). Since, the player has Rs.2 to start the play, the initial probability distribution of

x_0 is $p^{(0)} = (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0)$.

$$P^{(1)} = P^{(0)}, P = (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0) P = \left[0 \frac{1}{2} \ 0 \ \frac{1}{2} \ 0 \ 0 \ 0 \right]$$

$$P^{(2)} = P^{(1)}, P = \left(0 \frac{1}{2} \ 0 \ \frac{1}{2} \ 0 \ 0 \ 0 \right) P = \left[\frac{1}{4} \ 0 \ \frac{1}{2} \ 0 \ \frac{1}{4} \ 0 \ 0 \right]$$

$$P^{(3)} = P^{(2)}, P = \left(\frac{1}{4} \ 0 \ \frac{1}{2} \ 0 \ \frac{1}{4} \ 0 \ 0 \right) P = \left[\frac{1}{16} \ \frac{1}{4} \ 0 \ \frac{3}{8} \ 0 \ \frac{1}{8} \ 0 \right]$$

$$P^{(4)} = P^{(3)}, P = \left(\frac{1}{16} \ \frac{1}{4} \ 0 \ \frac{3}{8} \ 0 \ \frac{1}{8} \ 0 \right) P = \left[\frac{3}{256} \ 0 \ \frac{5}{16} \ 0 \ \frac{1}{4} \ 0 \ \frac{1}{16} \right]$$

$$P^{(5)} = P^{(4)}, P = \left[\frac{3}{256} \ 0 \ \frac{5}{16} \ 0 \ \frac{9}{32} \ 0 \ \frac{1}{8} \ 0 \right] P = \left[\frac{3}{256} \ \frac{5}{512} \ 0 \ \frac{9}{256} \ 0 \ \frac{1}{16} \ \frac{1}{64} \right]$$

The probability that the player has lost his money at the end of 5 plays = $P(x_5 = 0) = \frac{3}{8}$.

This value corresponds to the entry in state 0 of $P^{(5)}$.

- (c). The probability that the game lasts more than 7 plays = $P\{ \text{System is neither in state 0 nor in 6 at the end of seventh round} \}$.

$$\text{Using } P^{(6)} = P^{(5)}P = \left(\frac{29}{64} \ 0 \ \frac{7}{32} \ 0 \ \frac{13}{64} \ 0 \ \frac{1}{8} \right)$$

$$P^{(7)} = P^{(6)} \cdot P = \left(\frac{29}{64} \ 0 \ \frac{27}{128} \ 0 \ \frac{13}{128} \ 0 \ \frac{1}{8} \right)$$

$$P\{x_7 = 1, 2, 3, 4 \text{ (or) } 5\} = \frac{7}{64} + 0 + \frac{27}{128} + 0 + \frac{13}{128} = \frac{27}{64}$$

- ③ The transition probability matrix of a Markov chain $\{x_n\}; n = 1, 2, 3, \dots$ having three states 1, 2, 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$.

Find (i) $P\{x_2 = 3\}$, (ii) $P\{x_3 = 2, x_2 = 3, x_1 = 3, x_0 = 2\}$.

We have $P(x_0 = 1) = 0.7$; $P(x_0 = 2) = 0.2$,

$$P(x_0 = 3) = 0.1.$$

$$P^{(2)} = P^2 = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

finding vector

$$= \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{bmatrix}$$

$$(i) P\{x_2=3\} = \sum_{i=1}^3 P\{x_2=3/x_0=i\} \cdot P\{x_0=i\}$$

$$= P_{13}^{(2)} \cdot P\{x_0=1\} + P_{23}^{(2)} \cdot P\{x_0=2\} + P_{33}^{(2)} \cdot P\{x_0=3\}$$

and since there are 3 states, sum of total probabilities will be 1.

$$\text{and as } P\{x_0=i\} = 0.26 \times 0.7 + 0.34 \times 0.2 + 0.29 \times 0.1$$

$$= 0.182 + 0.068 + 0.029$$

$$= 0.279$$

$$(ii) P\{x_1=3/x_0=2\} = P_{23} = 0.2$$

$$P\{x_1=3, x_0=2\} = P\{x_1=3/x_0=2\} \times P\{x_0=2\} \\ = 0.2 \times 0.2 = 0.04 \text{ using } (i) - (ii)$$

$$P\{x_2=3/x_1=3, x_0=2\} = P\{x_2=3/x_1=3, x_0=2\} \times P\{x_1=3, x_0=2\}$$

$$\text{and similarly } P\{x_2=3/x_1=3\} \times P\{x_1=3, x_0=2\} \text{ by markov property.}$$

$$= 0.3 \times 0.04 \text{ by eqn (2)}$$

$$= 0.012$$

$$P\{x_3=2, x_2=3, x_1=3, x_0=2\}$$

$$= P\{x_3=2/x_2=3, x_1=3, x_0=2\} \times P\{x_2=3/x_1=3, x_0=2\}$$

$$= P\{x_3=2/x_2=3\} \times P\{x_2=3/x_1=3, x_0=2\} \text{ by markov property}$$

$$= 0.4 \times 0.012 \text{ by eqn (3)}$$

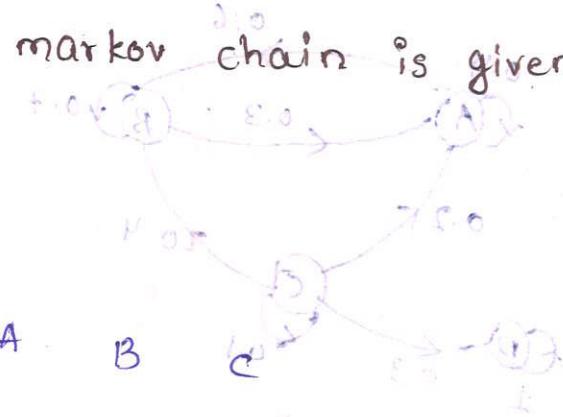
$$= 0.0048$$

$$= 0.0048$$

T P M Problems :-

① The three state markov chain is given by the dpm?

$$P = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$



Sol:- After 1000 steps
 $P = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

∴ In this markov chains; all the states communicate with each other.

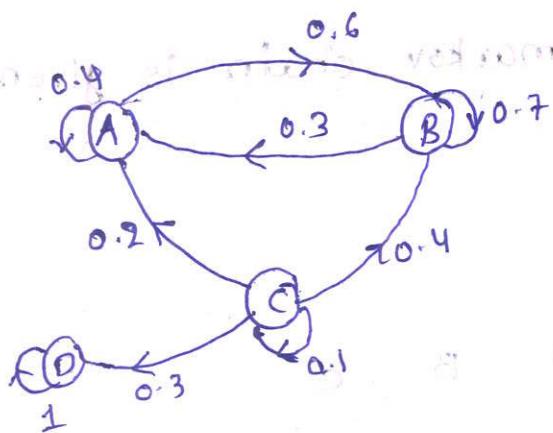
The chain is irreducible.

② Consider the markov chain with dpm ?

$$P = \begin{bmatrix} 0.4 & 0.6 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Sol:-

	A	B	C	D	E	F
A	0.4	0.6	0	0	0	0
B	0.3	0.7	0	0	0	0
C	0.2	0.4	0.1	0.3	0	0
D	0	0	0	0.1	0	0
E	0	0	0	0	0	0
F	0	0	0	0	0	0



All the states do not communicate with each other.

The chain is not irreducible (or) ergodic.

- ③ A fair die is tossed repeatedly? If x_n denotes the maximum of the numbers occurring in the first n tosses. find the transition probability matrix P of the markov chain $\{x_n\}$. Find P^2 and $P(x_2=6)$.

Soln:- Sample space $S = \{1, 2, 3, 4, 5, 6\}$.

The transition Probability matrix has the following entries in main diagonal
 \rightarrow no on face

$$P = \begin{bmatrix} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \text{trials} \uparrow & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \frac{5}{6} & \frac{4}{6} & \frac{3}{6} & \frac{2}{6} & \frac{1}{6} \\ 2 & 0 & 0 & \frac{4}{6} & \frac{3}{6} & \frac{2}{6} & \frac{1}{6} \\ 3 & 0 & 0 & 0 & \frac{3}{6} & \frac{2}{6} & \frac{1}{6} \\ 4 & 0 & 0 & 0 & 0 & \frac{2}{6} & \frac{1}{6} \\ 5 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^2 = \frac{1}{36} \begin{bmatrix} 1 & 3 & 5 & 7 & 9 & 11 \\ 0 & 4 & 5 & 7 & 9 & 11 \\ 0 & 0 & 9 & 7 & 9 & 11 \\ 0 & 0 & 0 & 16 & 9 & 11 \\ 0 & 0 & 0 & 0 & 25 & 11 \\ 0 & 0 & 0 & 0 & 0 & 36 \end{bmatrix}$$

(10)

Initial state Probability distribution is

$$P^{(0)} = \left[\frac{1}{6}, \frac{1}{16}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right]$$

$P(X_2=6) = P^2 (6^{\text{th}} \text{ column} \times P^{(0)})$:

$$\begin{aligned} &= \frac{1}{3} \left\{ \begin{bmatrix} 11 \\ 11 \\ 11 \\ 11 \\ 11 \\ 36 \end{bmatrix} \times \left(\frac{1}{6}, \frac{1}{16}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right) \right\} \\ &= \frac{1}{36} \times \frac{1}{6} (11 + 11 + 11 + 11 + 11 + 36) \\ &= \frac{91}{216} \end{aligned}$$

Note :-

A matrix is said to be stochastic matrix if it satisfies the following conditions.

- ① The given matrix should be square matrix
- ② All entries the matrix must be non-negative (positive) (include zero). That sum of the elements of each row is equal to 1.
- ③ The sum of all elements of the matrix is 1.

Note :-

conditions for regular stochastic matrix.

- ① The matrix should be stochastic matrix.
- ② The principle diagonal elements of the matrix given not column unit (1).
- ③ The power of the given matrix do not contain zero's (it mean all elements are positive).